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Modeling of General Surface Junctions of Composite Objects in an SIE/MoM Formulation

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Abstract This paper discusses the modeling of various kinds of surface junctions in an SIE/MoM (Surface Integral Equation / Method of Moments) formulation applied to complex objects consisting of arbitrarily shaped conducting and dielectric bodies. Methods of describing various types of junctions and systematically incorporating them in numerical solutions are presented. The procedures are of interest for the specific application of arbitrarily shaped dielectric resonator antennas and their associated feed structures and packaging. An E-PMCHW formulation in conjunction with a moment method procedure using generalized triangular basis functions is presented to deal with such general junctions.

I. Introduction

The modeling of general surface junctions in an SIE/MoM formulation is considered in this work. The specific application leading to this study is that of a dielectric resonator (DR) antenna. Since an experimental study of a cylindrical dielectric resonator (DR) antenna was reported in 1983 [1], this antenna has drawn continued interest because of its small size, efficiency, and potential ability to perform multiple antenna tasks via simple mode coupling mechanisms. The configuration of a DR antenna may range from a very simple one which allows analytic solutions to a very complex one. A typical structure for a DR antenna is a DR element of high dielectric constant excited by a single feed such as a microstripline or coaxial cable. Various shapes and combinations of DR elements as well as various feed structures have been suggested, however, which may improve the antenna performance in the areas of bandwidth, power handling, and antenna efficiency.

Rigorous SIE analysis methods for non-trivial DR antenna configurations have been available mainly for body of revolution (BOR) objects [2,3]. This work results from an interest in the analysis of DR antennas of more arbitrary configurations, which may include general 3D objects comprising an arbitrary combination of conducting and/or dielectric bodies of arbitrary shapes, using an SIE/MoM method with triangular patch basis functions. The junction modeling problem has been considered in previous works for conducting surfaces [4], for dielectric surfaces [5], simple combinations of both for BOR objects [2,3,6,7], and for a general case of conducting, dielectric, resistive, and impedance boundary condition surfaces [8]. This work attempts to provide a formalism for systematically describing junction models for a wide variety of junction types.

II. Formulation

A. Problem Description

The geometry under consideration is a general inhomogeneous body with N_R dielectric regions, each of which may contain conducting bodies as well as impressed sources as shown in Fig. 1.

The regions have permittivities ϵ_i and permeabilities μ_i, where $i = 1, \ldots, N_R$. Both ϵ_i and μ_i may be complex to represent lossy materials. Non-zero thickness conducting bodies denoted by R_0 may occupy any parts of the space. Infinitely thin conducting bodies can reside in any region, at interfaces between regions, or they may penetrate from one region to another. All conductors are considered to be PEC (Perfect Electric Conductor) material. One of the regions, region R_1 in Fig. 1, may be of infinite extent. The total fields in each region are denoted by $\mathbf{E}_i$ and $\mathbf{H}_i$, where $i = 0, 1, 2, \ldots, N_R$, for electric and magnetic fields, respectively, and $i = 0$ denotes PEC regions with $\mathbf{E}_0 = \mathbf{H}_0 = 0$. The time variation, $e^{j\omega t}$, is assumed and suppressed throughout.

Any two adjacent regions, R_i and R_j, are separated by a surface denoted by $S_{ij}(t_s, t, f)$, where t_s is the type of the surface, and t and f are the 'to-region' and the 'from-region' of the surface, respectively, which define the region connectivity and the surface orientation. The interface between a non-zero thickness conducting body and a dielectric region also forms a surface denoted in the same way with the 'from-region' being region zero. An infinitely thin conducting body in a dielectric region forms yet another type of surface with the 'from-region' being the same as the 'to-region. Thus, as shown in Fig. 1, there are four types of surfaces specified by t_s —

- Type-0 ($t_s = 0$, pf0) ; interface between a conducting body and a dielectric region,
- Type-1 ($t_s = 1$, pf1) ; infinitely thin conducting body within a dielectric region,
- Type-2 ($t_s = 2$, pf2) ; infinitely thin conducting body between two dielectric regions, and
- Type-3 ($t_s = 3$, df) ; dielectric interface between two dielectric regions.

When more than two surfaces meet at a curved line segment, they form a junction. Depending on the numbers and types of the surfaces at a junction, there are a variety of possible junction types, all of which are considered in this study.

Each region R_i is surrounded by a closed surface S_i^C and is associated with an inward normal unit vector $\hat{\mathbf{n}}_i$. The surface interface between regions R_i and R_j, if one exists, is denoted as S_{ij}, for any i and j, $i = 1, \ldots, N_R$, $j = 0, 1, \ldots, N_R$. Thus, S_i^C is the set of all interface surfaces S_{ij}, where j represents all region numbers that interface with region R_i. Note that $S_{ij} = S_{ji}$ for $j \neq 0$; however, the normal unit vectors $\hat{\mathbf{n}}_i$ and $\hat{\mathbf{n}}_j$ are in opposite directions to each other on S_{ij}.

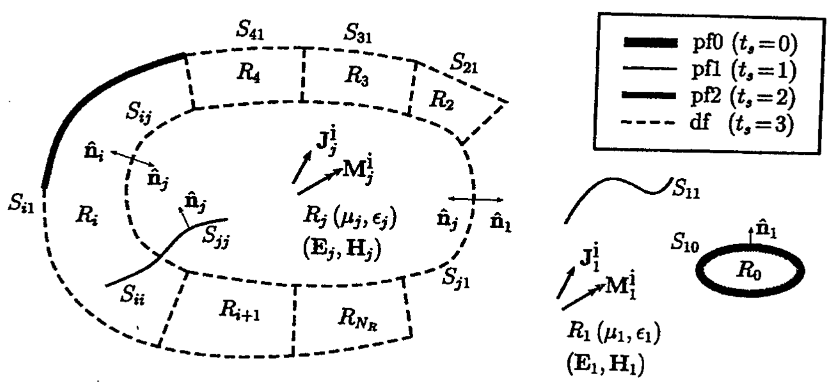


Fig. 1. General geometry under consideration.

B. The Field Equivalences

According to the equivalence principle [9], the original problem can be decomposed into N_R auxiliary problems, one for each dielectric region. To obtain the auxiliary problem for region R_i, the impressed sources of the original problem are retained only in region R_i and the boundaries of the region are replaced by equivalent surface currents radiating in a homogeneous medium with the constitutive parameters of region R_i. Electric currents are used for the conducting surfaces, while electric and magnetic currents are used for the dielectric boundaries. The electric and magnetic currents appearing on opposite sides of a dielectric interface in different auxiliary problems are taken equal in magnitude and opposite in direction to assure the continuity of the tangential components on these boundaries as they are continuous in the original problem. In this procedure, the fields produced within the region boundaries by the equivalent currents and the impressed sources in region R_i must be the same as those in the original problem, while the zero field is produced outside these boundaries. The electric and magnetic currents along S_i^C are $\mathbf{J}_i = \hat{\mathbf{n}}_i \times \mathbf{H}_i$ and $\mathbf{M}_i = \mathbf{E}_i \times \hat{\mathbf{n}}_i$, respectively.

A system of surface integro-differential equations can be obtained by enforcing the boundary conditions of continuity of the tangential components of electric field on the conducting surfaces and both electric and magnetic fields on the dielectric surfaces. This results in the E-PMCHW formulation [6] when there is no junction in the problem. For problems having general junctions, however, it is not easy to express the integral equation system explicitly apart from the testing procedure. Thus the system of integral equations is presented in the next section after describing the junction modeling and the basis functions.

C. Modeling of Junctions in the Moment Method Solution

Arbitrarily shaped surfaces are discretized in triangular patches and the equivalent surface currents are approximated by expansions in the RWG basis functions on the patches, which are expressed as [10]

$$\mathbf{J}(\mathbf{r}) \cong \sum_{n=1}^{N_{T_j}} I_n \mathbf{B}_n^{T_j}(\mathbf{r}; S_{T_n}^+, S_{T_n}^-) \tag{1}$$

where,

$$\mathbf{B}_n^{T_j}(\mathbf{r}) = \begin{cases} \pm \rho_n^\pm / h_n^\pm, & \mathbf{r} \in S_{T_n}^\pm \\ 0, & \text{otherwise,} \end{cases} \tag{2}$$

N_{T_j} is the number of electric basis functions, and $S_{T_n}^\pm$ are the positive/negative domains or the from-/to- faces of the basis function, respectively. For magnetic currents, $\{\mathbf{B}_n^{T_m}\}_{n=1}^{N_{T_m}}$ can be defined similarly. The testing functions $\mathbf{T}_n^{T_j}$ and $\mathbf{T}_n^{T_m}$ are also taken to be the same as (2). With the basis and testing functions defined we have a matrix equation

$$\begin{bmatrix} \mathbf{Z}^{T_j T_j} & \mathbf{T}^{T_j T_m} \\ \mathbf{T}^{T_m T_j} & \mathbf{Y}^{T_m T_m} \end{bmatrix} \begin{bmatrix} |I^{T_j}\rangle \\ |I^{T_m}\rangle \end{bmatrix} = \begin{bmatrix} |V^{T_j}\rangle \\ |V^{T_m}\rangle \end{bmatrix} \tag{3}$$

When there are general surface junctions, the current related to an unknown coefficient may exist on many different surfaces. In such cases, the expression (1) is not rigorous enough. For example, there is an electric current on a dielectric surface in the region R_i equivalent problem and another one flowing in the opposite direction in the region R_j problem, represented by '$-I_n$'

as shown in Fig. 2(a). The expression in (1) for the electric currents has this sort of implication for the basis functions $\mathbf{B}_n^{T_j}$ when the domain of the unknown involves a dielectric interface. When more than two dielectric surfaces meet at a junction, this scheme does not work. Thus for general junctions, we seek another way of expressing the generalized current more rigorously. We will use two different basis functions for the same unknown coefficient related to a dielectric surface as shown in Fig. 2(b). In other words, the unknown coefficient has a multiplicity of two when it represents the electric or magnetic current on the dielectric face. Extending this to the general case, the generalized current is defined in terms of the generalized basis functions as

$$\mathbf{C}(\mathbf{r}) = \{\mathbf{J}(\mathbf{r}), \mathbf{M}(\mathbf{r})\} = \{\sum_{n=1}^{N_{T_j}} I_n \mathbf{B}_n(\mathbf{r}), \sum_{n=1+N_{T_j}}^{N} I_n \mathbf{B}_n(\mathbf{r})\} \tag{4}$$

where,

$$\mathbf{B}_n(\mathbf{r}) = \mathbf{B}_k^{T_j}(\mathbf{r}),\ \mathbf{B}_{n_v}(\mathbf{r}) = \mathbf{B}_{k_v}^{T_j}(\mathbf{r}),\ \tau_n = \tau_k, \quad \text{with } k = n, \qquad \text{if } n \le N_{T_j} \tag{5}$$

$$\mathbf{B}_n(\mathbf{r}) = \mathbf{B}_k^{T_m}(\mathbf{r}),\ \mathbf{B}_{n_v}(\mathbf{r}) = \mathbf{B}_{k_v}^{T_m}(\mathbf{r}),\ \tau_n = \tau_k, \quad \text{with } k = n - N_{T_j}, \qquad \text{if } n > N_{T_j} \tag{6}$$

$$N = N_{T_j} + N_{T_m}$$

$$\mathbf{B}_k^{T_j}(\mathbf{r}) = \sum_{v=1}^{\tau_k} \mathbf{B}_{k_v}^{T_j}(\mathbf{r}; ff_{k_v}, tf_{k_v}, R_{k_v}) \tag{7}$$

$$\mathbf{B}_k^{T_m}(\mathbf{r}) = \sum_{v=1}^{\tau_k} \mathbf{B}_{k_v}^{T_m}(\mathbf{r}; ff_{k_v}, tf_{k_v}, R_{k_v}) \tag{8}$$

$\mathbf{B}_{n_v}$ = the v^{th} basis function of I_n, $v = 1, ..., \tau_n$

$\mathbf{B}_{k_v}^{T_j}, \mathbf{B}_{k_v}^{T_m}$ = RWG basis function defined over the corresponding patches as in (2)

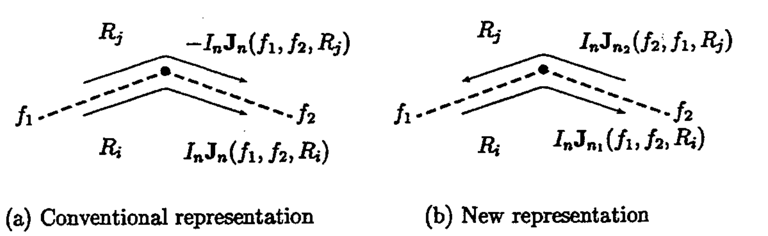
τ_n = multiplicity of the unknown coefficient, $I_n = \begin{cases} n_{dfn}, & n_{dfn} = n_{tf} \\ n_{dfn} + 1, & \text{otherwise} \end{cases}$

n_{tf} = total number of faces (surfaces) connected to the junction for I_n

n_{dfn} = number of dielectric faces (surfaces) related to I_n

ff_{n_v}, tf_{n_v} = from-face and to-face of $\mathbf{B}_{n_v}$.

R_{n_v} = region of $\mathbf{B}_{n_v}$.



(a) Conventional representation (b) New representation

Fig. 2. Two methods of representing basis functions.

Notice that there is one-to-one correspondence between $\mathbf{B}_{n_v}^{T_j}$ or $\mathbf{B}_{n_v}^{T_m}$ and the parameter set $\{ff_{n_v}, tf_{n_v}, R_{n_v}\}$. The numbers of unknowns and basis functions for a given junction or edge are determined from the types and numbers of the faces connected to the junction by considering proper boundary conditions at the junction. The methods of determining them and systematically incorporating them in the MoM solutions have been developed. Examples of modeling general junctions are shown in Fig. 3, where J_n and M_n are used instead of $\mathbf{B}_n^{T_j}$ and $\mathbf{B}_n^{T_m}$, respectively. The generalized testing functions $\{\mathbf{T}_m^{T_j}\}_{m=1}^{N_{T_j}}$, $\{\mathbf{T}_m^{T_m}\}_{m=1}^{N_{T_m}}$, and $\{\mathbf{T}_m\}_{m=1}^{N}$ are also defined in a similar manner. We also define $\mathbf{C}_i$, the generalized current for the region R_i equivalent problem, as

$$\mathbf{C}_i(\mathbf{r}) = \{\mathbf{J}_i(\mathbf{r}), \mathbf{M}_i(\mathbf{r})\} \tag{9}$$

where

$$\mathbf{J}_i(\mathbf{r}) = \sum_{n=1}^{N_{T_j}} I_n \sum_{v=1}^{\tau_n} \delta_{n_v i}^S \mathbf{B}_{n_v}(\mathbf{r}; ff_{n_v}, tf_{n_v}, R_{n_v}) \tag{10}$$

$$\mathbf{M}_i(\mathbf{r}) = \sum_{n=N_{T_j}+1}^{N} I_n \sum_{v=1}^{\tau_n} \delta_{n_v i}^S \mathbf{B}_{n_v}(\mathbf{r}; ff_{n_v}, tf_{n_v}, R_{n_v}) \tag{11}$$

$$\delta_{n_v i}^S = \text{source contribution coefficient} = \begin{cases} 1, & R_{n_v} = R_i \\ 0, & \text{otherwise} \end{cases}$$

With the set of basis functions in (4–8), one may apply the boundary conditions of tangential field continuity at each sub-domain of the basis functions. By merely applying the boundary conditions, however, the total number of equations may be greater than the number of the unknowns because of the multiplicity of some unknowns related to junctions. The usual methods of solving equations apply only when the number of equations equals to the number of unknowns, N. Such a set of N equations can be obtained by taking the n^{th} integral equation as the set of simultaneous integral equations (or summation of them) which satisfy the proper boundary conditions on the subdomains of the basis functions ($\mathbf{B}_{n_v}, v = 1, ..., \tau_n$) related to the unknown coefficient, I_n. It is possible to get such a surface integral equation system by testing with the generalized testing functions as follows

$$\sum_{i=1}^{N_R} \langle \mathbf{E}_i^{\mathrm{S}}(\mathbf{C}_i)|_{\tan}, \sum_{u=1}^{\tau_m} \delta_{m_u i}^F \, \mathbf{T}_{m_u} \rangle = -\sum_{i=1}^{N_R} \langle \mathbf{E}_i^{\mathrm{i}}|_{\tan}, \sum_{u=1}^{\tau_m} \delta_{m_u i}^F \, \mathbf{T}_{m_u} \rangle, \; m = 1, 2, ..., N_{T_j} \tag{12}$$

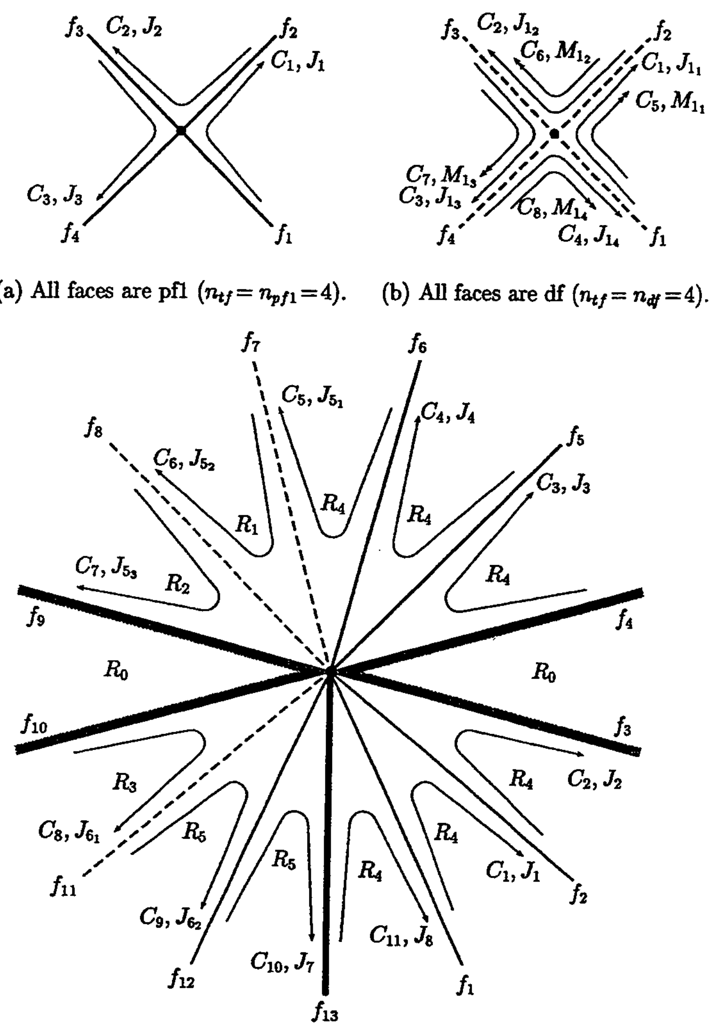
$$\sum_{i=1}^{N_R} \langle \hat{\mathbf{n}}_i \times \mathbf{H}_i^{\mathrm{S}}(\mathbf{C}_i), \sum_{u=1}^{\tau_m} \delta_{m_u i}^F \, \mathbf{T}_{m_u} \rangle = -\sum_{i=1}^{N_R} \langle \hat{\mathbf{n}}_i \times \mathbf{H}_i^{\mathrm{i}}, \sum_{u=1}^{\tau_m} \delta_{m_u i}^F \, \mathbf{T}_{m_u} \rangle, \quad m = N_{T_j} + 1, ..., N_{T_j} + N_{T_m}, \tag{13}$$

where,

$$\langle \mathbf{f}, \mathbf{g} \rangle = \int_S \mathbf{f} \cdot \mathbf{g} \, ds$$

$$\delta_{m_u i}^F = \text{field contribution coefficient} = \begin{cases} 1, & R_i = R_{m_u} \\ 0, & \text{otherwise} \end{cases}$$

$$R_{m_u} = \text{region of the testing function, } \mathbf{T}_{m_u},$$



(a) All faces are pf1 ($n_{tf} = n_{pf1} = 4$). (b) All faces are df ($n_{tf} = n_{df} = 4$).

(c) A general case ($n_{tf} = 13, n_{pf0} = 4, n_{pf1} = 5, n_{pf2} = 1, n_{df} = 3$).

Fig. 3. Modeling of general surface junctions. ($C_i, i = 1, 2, 3, \ldots$, is an entry-counting index.)

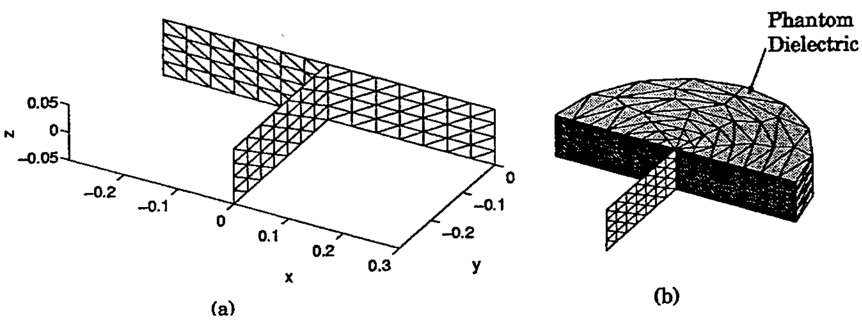


Fig. 4. A test case. (a) PEC alone (b) with phantom dielectric.

and ($\mathbf{E}_i^{\mathrm{S}}$, $\mathbf{H}_i^{\mathrm{S}}$) and ($\mathbf{E}_i^{\mathrm{I}}$, $\mathbf{H}_i^{\mathrm{I}}$) are the scattered fields due to $\mathbf{C}_i$ and incident fields, respectively. Equations (12) and (13) are the E-PMCHW formulation [6] extended to general junctions. Substituting $\mathbf{C}_i$ into (12) and (13), the impedance matrix and excitation vector elements in (3), $Z_{mn}^{T_jT_j}$ and $V_m^{T_j}$, for example, are expressed as

$$Z_{mn}^{T_jT_j} = \sum_{i=1}^{N_R} \langle\, [\mathbf{E}_i^{\mathrm{S}}(\sum_{v=1}^{\tau_n} \delta_{n_v i}^{S} \mathbf{B}_{n_v}(\mathbf{r}'; ff_{n_v}, tf_{n_v}, R_{n_v}))]_{\tan}, \sum_{u=1}^{\tau_m} \delta_{m_u i}^{F} \mathbf{B}_{m_u}(\mathbf{r}; ff_{m_u}, tf_{m_u}, R_{m_u})\,\rangle,$$
$$n = 1, ..., N_{T_j} \ \text{and} \ m = 1, ..., N_{T_j} \quad (14)$$

$$V_m^{T_j} = -\sum_{i=1}^{N_R} \langle\, \mathbf{E}_i^{\mathrm{I}}|_{\tan}, \sum_{u=1}^{\tau_m} \delta_{m_u i}^{F} \mathbf{B}_{m_u}(\mathbf{r}; ff_{m_u}, tf_{m_u}, R_{m_u})\,\rangle, \quad m = 1, ..., N_{T_j} \quad (15)$$

Some subroutines of EMPACK [11] have been used for the integrations over the triangular domains which appear in (14) implicitly.

III. Numerical Results

A T-shape junction of three 0.1-m wide and 0.3-m long PEC strips is taken as an example. For comparison a semi-circular cylinder of phantom dielectric having 0.1-m height and 0.3-m radius is attached to the T-shape junction as shown in Fig. 4. The z-directed surface currents along the contour lines, $(-0.3, 0, 0) \rightarrow (0.3, 0, 0)$ and $(0, 0, 0) \rightarrow (0, -0.3, 0)$, located at the center of each strip are computed for a plane wave excitation. The plane wave is expressed as $\mathbf{E}^{inc} = E_o\, e^{k_o \hat{k}^i \cdot \mathbf{r}}$, where, $\hat{k}^i = -\hat{x}\cos\phi^i \sin\theta^i - \hat{y}\sin\phi^i \sin\theta^i - \hat{z}\cos\theta^i$, $E_o = E_\theta^i(\hat{x}\cos\theta^i\cos\phi^i + \hat{y}\cos\theta^i\sin\phi^i - \hat{z}\sin\theta^i)$, $\theta^i = \phi^i = 45°$, $E_\theta^i = 1$, $k_o = 2\pi f\sqrt{\mu_o\epsilon_o}$, and $f = 300$ MHz. The results in Fig. 5 show very good agreement as well as the expected singularities at the end of the strips.

IV. Conclusion

A systematic procedure for modeling of the general junctions of any combination of conducting and/or dielectric bodies in an SIE/MoM formulation has been presented. With the successful modeling of general junctions, it is possible to apply the E-PMCHW formulation to a large class of problems including dielectric resonator antennas of complex configuration.

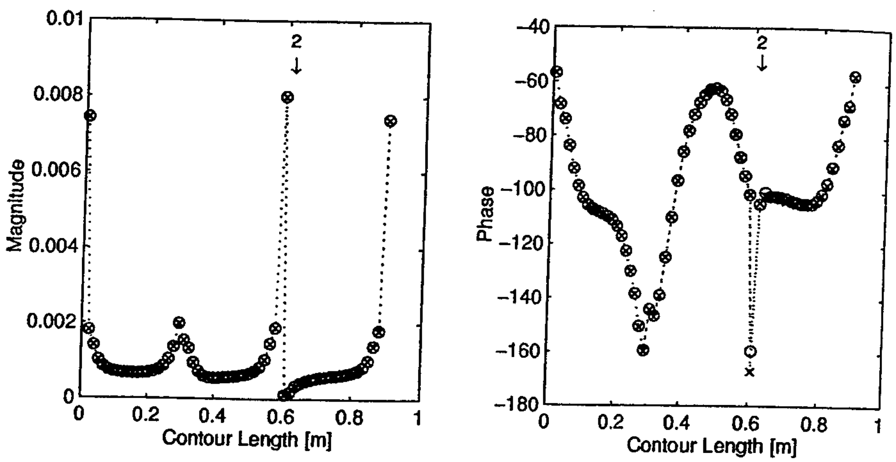


Fig. 5. z-directed current densities along the contours (∘ ; PEC alone, × ; with phantom dielectric). The arrows denote the start of the second contour.

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